If $\omega \rightarrow$ frequency of typical plasma oscillations and τ is mean time between collisions with neutral atoms, we require $\omega\tau > 1$ for the gas to behave like a plasma rather than a neutral gas.

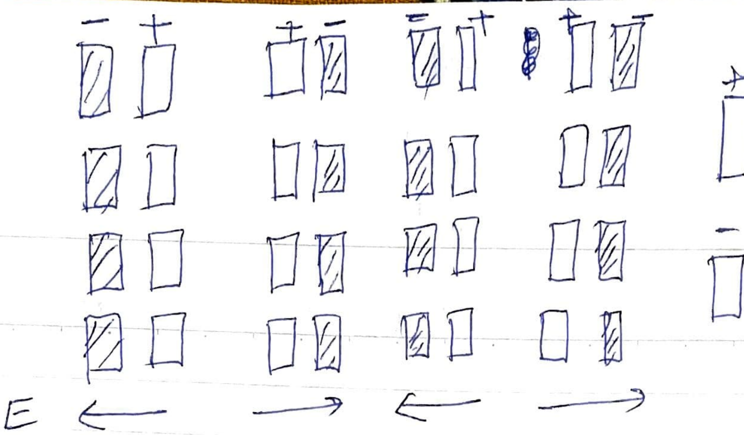
Three conditions a plasma must satisfy are

1. $\lambda_D << L$
2. $N_D >>> 1$
3. $\omega\tau > 1$

Plasma Oscillation:- If the electrons in a plasma are displaced from a uniform background of ions $\rightarrow$ electric field is built up in such a direction to restore the neutrality of plasma by pulling back the electrons to their original positions.

Because of their inertia $\rightarrow$ electrons overshoot and oscillate around their equilibrium positions with a characteristic frequency $\rightarrow$ Plasma frequency

Oscillations so fast $\rightarrow$ massive ions do not have time to respond to oscillating field $\rightarrow$ may be considered fixed



Mechanism of plasma oscillations

Plasma frequency ω_p

Assumptions (1) There is no magnetic field
(2) There are no thermal motions ($KT=0$)
(3) The ions are fixed in space in uniform distribution
(4) The plasma is infinite in extent
(5) The electron motion occur only in x-direction

From assumption (5), we have

$$\nabla = \hat{x}\frac{\partial}{\partial x} \qquad E = E\hat{x} \qquad \nabla \times E = 0 \qquad E = -\nabla\phi \quad --(1)$$

$\Rightarrow$ There is no fluctuating magnetic field, this is an electrostatic oscillation

The electron eq^{n}s of motion and continuity are

$m\frac{dv}{dt} = q(E + v\times B)$
fluid eq^n
$mn\frac{dv}{dt} = qn(E + v\times B)$
in case of plasma
$mn\left[\frac{\partial u}{\partial t} + (u\cdot\nabla)u\right] = qn(E + u\times B)$

$$mn_e\left[\frac{dv_e}{dt} + (v_e\cdot\nabla)v_e\right] = -en_eE \quad --(2)$$

$$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e v_e) = 0 \quad --(3)$$

Poisson's eq^n

Poisson's eq^n → that does not involve B
High frequency oscillations electron inertia is important, deviation from neutrality is main effect

$$\varepsilon_0 \nabla\cdot E = \varepsilon_0\frac{\partial E}{\partial x} = e(n_i - n_e) \quad --(4)$$

Amplitude of oscillation is small, terms involving higher powers of amplitude factors can be neglected

Seperate the dependent variables into two parts, 20

equilibrium part $\rightarrow$ 0

perturbation part $\rightarrow$ 1

$$n_e = n_0 + n_1 , \quad v_e = v_0 + v_1 , \quad E = E_0 + E_1 \quad \text{---}(5)$$

equilibrium quantities $\rightarrow$ state of plasma is the absence of the oscillations.

We have assumed a uniform neutral plasma at rest before the electrons are displaced, we have

$$\nabla n_0 = v_0 = E_0 = 0$$

$$\frac{\partial n_0}{\partial t} = \frac{\partial v_0}{\partial t} = \frac{\partial E_0}{\partial t} = 0 \quad \text{---}(6)$$

Eq^n (2) now becomes

$$m\left[\frac{\partial v_1}{\partial t} + \cancel{(v_1 \cdot \nabla) v_1}^{\,0}\right] = -eE_1 \quad \text{---}(7)$$

$(v_1 \cdot \nabla) v_1 \rightarrow$ quadratic in an amplitude quantity

linearize by neglecting it.

Similarly eq^n (3) becomes

$$\frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 v_1 + \cancel{n_1 v_1}) = 0 \quad \text{--}(8)$$

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot v_1 + \cancel{v_1 \cdot \nabla n_0} = 0$$

In Poisson's eq^n [4], we note that $n_{i0} = n_{e0}$ in equilibrium and that $n_{i1} = 0$ by the assumption of fixed ions, so we have

$$\varepsilon_0 \nabla \cdot E_1 = -en_1 \quad \text{---}(9)$$

The oscillating quantities are assumed to behave sinusoidally:

$$v_1 = v_1 e^{i(kx-\omega t)}\hat{x}$$
$$n_1 = n_1 e^{i(kx-\omega t)} \quad \text{--- (10)}$$
$$E = E e^{i(kx-\omega t)}\hat{x}$$

time derivative can be replaced by $\frac{\partial}{\partial t} \rightarrow -i\omega$ and the gradient ∇ by $ik\hat{x}$.

eq^n (7) – (9), now become

$$-im\omega v_1 = -eE_1 \quad \text{--- (11)}$$
$$-i\omega n_1 = -n_0 ikv_1 \quad \text{--- (12)}$$
$$ik\varepsilon_0 E_1 = -en_1 \quad \text{--- (13)}$$

Eliminating n_1 and E_1, we have for eq^n (11)

$$-im\omega v_1 = -e\frac{-e}{ik\varepsilon_0}\frac{-n_0ikv_1}{-i\omega} = -i\frac{n_0e^2}{\varepsilon_0\omega}v_1 \quad \text{--- (14)}$$

If v_1 does not vanish, we must have

$$\omega^2 = \frac{n_0e^2}{m\varepsilon_0}$$

Plasma frequency

$$\boxed{\omega_p = \left(\frac{n_0e^2}{\varepsilon_0 m}\right)^{1/2} \text{rad/sec}} \quad \text{--- (15)}$$

Numerically we can use approximate formula

$$\omega_p/2\pi = f_p \approx 9\sqrt{n} \quad \text{--- (16)}$$

frequency → depending only on the plasma density → one of the fundamental parameter of plasma

n → small, usually f_p → very high

Ex: in a plasma of density $n = 10^{18}\ m^{-3}$

$$f_p \simeq 9(10^{18})^{1/2} = 9 \times 10^{9}\ sec^{-1}$$

$$= 9\ GHz$$

In particular, ω does not depend on k, so the group velocity $\frac{d\omega}{dk}$ is zero. The disturbance does not propagate.

Electron plasma Waves

Thermal motion:— Another effect that can cause plasma oscillations to propagate

Electrons streaming into adjacent layers of plasma with their thermal velocities will carry information about what is happening in the oscillating region.

Plasma oscillation —— can be called a plasma wave.

This effect can be treated by adding a term $-\nabla p_e$ to the eqn of motion

In the one dimensional problem, γ will be 3,

Hence

$$\nabla p_e = 3 kT_e \nabla n_e$$

$$= 3 KT_e \nabla(n_0 + n_1)$$

$$= 3KT_e \frac{\partial n_1}{\partial x}\hat{x}$$

and the linearized eqn of motion is

$$m n_0 \frac{\partial v_1}{\partial t} = -e n_0 E_1 - 3kT_e \frac{\partial n_1}{\partial x} \quad \text{---(17)}$$

in linearizing we have neglected the terms $n_1 \frac{\partial v_1}{\partial t}$, and $n_1 E_1$ as well as $(v_1 \cdot \nabla) v_1$ term.

With eqn (10) eqn (17) becomes

$$-im\omega n_0 v_1 = -e n_0 E_1 - 3kT_e ik n_1 \quad \text{---(18)}$$

n_1, E_1 given by eqn (12) & (13) and we have

$$im\omega n_0 v_1 = \left[+ e n_0 \left(\frac{-e}{ik\varepsilon_0} \right) + 3KT_e ik \right] \frac{n_0 ik}{i\omega} v_1$$

$$\omega^2 v_1 = \left(\frac{n_0 e^2}{\varepsilon_0 m} + \frac{3kT_e}{m} k^2 \right) v_1$$

$$\boxed{\omega^2 = \omega_p^2 + \frac{3}{2} k^2 v_{th}^2} \quad \text{---(19)}$$

where $v_{th}^2 = \frac{2kT_e}{m}$

The frequency now depends on k, and the group velocity is finite.

$$2\omega d\omega = \frac{3}{2} v_{th}^2 \, 2k dk$$

$$v_g = \frac{d\omega}{dk} = \frac{3}{2} \frac{k}{\omega} v_{th}^2 = \frac{3}{2} \frac{v_{th}^2}{v_\phi}$$

Sound waves: Theory of sound waves in ordinary air

Neglecting viscosity, we can write the Navier-Stokes eq^n as $\rightarrow \rho \eta \nabla^2 v$

$$\rho\left[\frac{\partial v}{\partial t}+(v\cdot\nabla)v\right]=-\nabla p=-\frac{\gamma p}{\rho}\nabla\rho \quad \text{---}(1)$$

The eq^n of continuity is

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho v)=0 \quad \text{---}(2)$$

linearizing about a stationary equilibrium with uniform p_0 and ρ_0, we have

$$-i\omega\rho_0 v_1=-\frac{\gamma p_0}{\rho_0}ik\rho_1 \quad \text{---}(3)$$

$$-i\omega\rho_1+\rho_0 ik\cdot v_1=0 \quad \text{---}(4)$$

we have taken a wave dependence of the form $\exp[i(k\cdot r-\omega t)]$

For a plane wave with $k=k\hat{x}$ and $v=v\hat{x}$, we find, upon eliminating ρ_1,

$$-i\omega\rho_0 v_1=-\frac{\gamma p_0}{\rho_0}ik\frac{\rho_0 ikv_1}{i\omega}$$

$$\omega^2 v_1=k^2\frac{\gamma p_0}{\rho_0}v_1$$

or $$\frac{\omega}{k}=\left(\frac{\gamma p_0}{\rho_0}\right)^{1/2}=\left(\frac{\gamma KT}{M}\right)^{1/2}\equiv C_s \quad \text{---}(5)$$

$C_s \rightarrow$ expression for velocity of sound waves in a neutral gas. The waves are pressure waves propagating from one layer to the next by collisions among the air molecules.

In plasma $\rightarrow$ analogous phenomenon $\rightarrow$ ion acoustic wave or ion wave